Period 4, Nov 7, 2024

Solve the quadratic equation by completing the square.

$x^2 + 6x = -8 \Rightarrow x^2 + 6x + 9 = -8 + 9$

$a = 1$

$b = 6$

$\left(\frac{b}{2}\right) = \left(\frac{6}{2}\right) = 3$

$\left(\frac{b}{2}\right)^2 = (3)^2 = 9$

$(x+3)^2 = 1$

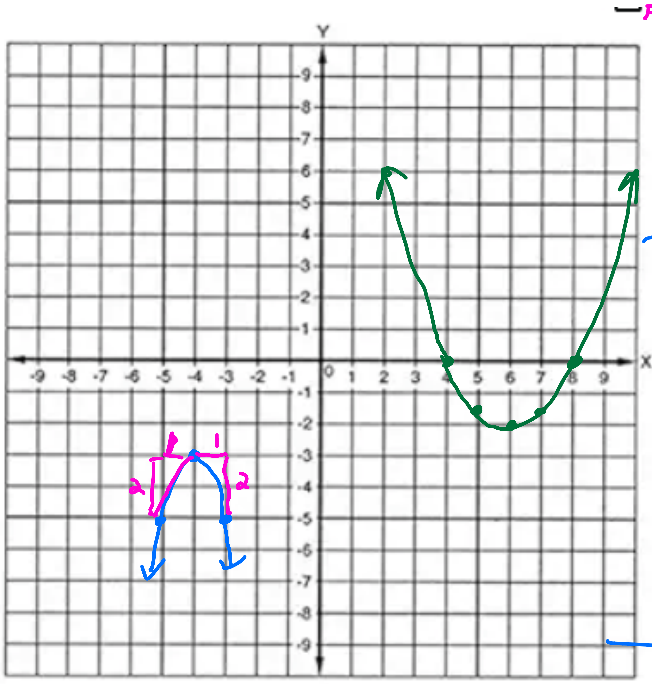
$\sqrt{(x+3)^2} = \sqrt{1}$

$|x+3| = 1$

$x + \cancel{3} = 1$ or $x + \cancel{3} = -1$

$\cancel{-3} \quad -3 \qquad \cancel{-3} \quad -3$

$x = -2$ or $x = -4$



Scale Factor

$y = \pm a(x-h)^2 + k$

Vertex (h,k)

$+$ opens up

$-$ opens down

$y = -2(x+4)^2 - 3$

open down

vertex $(-4,-3)$

x	y
-3	$-5 = -2(1)^2 - 3$
-5	$-5 = -2(-1)^2 - 3$
-4	-3

$y = \frac{1}{2}(x-6)^2 - 2$

vertex $= (6,-2)$

open up

x	y
6	-2
5	$-1\frac{1}{2}$
7	$-1\frac{1}{2}$
8	$0 = \frac{1}{2}\cdot 4 - 2$
4	$0 = \frac{1}{2}\cdot 4 - 2$
10	$6 = \frac{1}{2}\cdot 16 - 2$
2	$6 = \frac{1}{2}(-4)^2 - 2$

The graph of a quadratic function is given. Match the equation with its graph.

$j(x) = (x-4)^2 - 4$	$h(x) = (x-4)^2 + 4$	$f(x) = (x+4)^2 - 4$	$g(x) = (x+4)^2 + 4$
Vertex $(4,-4)$ opens up Scale of 1	Vertex $(4,4)$ opens up	Vertex $(-4,-4)$ opens up	Vertex $(-4,4)$ opens up

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Find the coordinates of the vertex for the parabola defined by the given quadratic function.

$f(x) = 2x^2 - 16x + 1$

$a=2$

$2(x^2 - 8x) + 1$

$a=1$

$b=-8$

$\frac{b}{a} = -4$

$\left(\frac{b}{a}\right)^2 = 16 \Rightarrow$

$2(\underbrace{x^2 - 8x + 16} - 16) + 1$

$2((x-4)^2 - 16) + 1$

$2(x-4)^2 - 32 + 1 = 2(x-4)^2 - 31$

Vertex $(4,-31)$

$\frac{-b}{2a}$ = x part of vertex

$F(x) = 2x^2 - 16x + 1$

$a=2$

$b=-16$

$\frac{-(-16)}{2 \cdot 2} = \frac{16}{4} = 4 \Rightarrow F(4) = 2(4)^2 - 16(4) + 1$

$2 \cdot 16 - 64 + 1$

$32 - 64 + 1 = -31$

Find the coordinates of the vertex for the parabola defined by the given quadratic function.

$f(x) = -x^2 + 6x + 9$

Complete The Square

$-[x^2 - 6x] + 9$

$a = 1$
$b = -6$
$\frac{b}{2} = -3$
$\left(\frac{b}{2}\right)^2 = 9$

$-[\underbrace{x^2 - 6x + 9} - 9] + 9$

$-[(x-3)^2 - 9] + 9$

$-(x-3)^2 + 9 + 9 \Rightarrow F(x) = -(x-3)^2 + 18$

vertex $(3, 18)$

$-\frac{b}{2a} = x$ Value of vertex

$\frac{-6}{2(-1)} = \frac{-6}{-2} = 3$

$F(3) = -(3)^2 + 6(3) + 9$

$-9 + 18 + 9 = 18$

$\Big\}$ vertex $(3, 18)$

Find the difference quotient of f; that is, find $\frac{f(x+h) - f(x)}{h}$, $h \neq 0$, for the following function. Be sure to simplify.

$f(x) = 2x^2 + x + 3$

$F(x+h) = 2(x+h)^2 + (x+h) + 3$

$2(x^2 + 2xh + h^2) + (x+h) + 3$

$2x^2 + 4xh + 2h^2 + x + h + 3$

$\frac{2x^2 + 4xh + 2h^2 + x + h + 3 - (2x^2 + x + 3)}{h}$

$\frac{\cancel{2x^2} + 4xh + 2h^2 + \cancel{x} + h + \cancel{3} - \cancel{2x^2} - \cancel{x} - \cancel{3}}{h} = \frac{4xh + 2h^2 + h}{h} = \frac{\cancel{h}(4x + 2h + 1)}{\cancel{h}}$

$4x + 2h + 1 = \frac{F(x+h) - F(x)}{h}$

Use the vertex and intercepts to sketch the graph of the quadratic function. Give the equation for the parabola's axis of symmetry. Use the graph to determine the function's domain and range.

$f(x) = 6x^2 + 12x - 1$

$y = 6x^2 + 12x - 1$

$y = 6(x^2 + 2x + 1 - 1) - 1 \Rightarrow y = 6[(x+1)^2 - 1] - 1$

$a = 1$

$b = 2$

$\frac{b}{2} = 1$

$(\frac{b}{2})^2 = (1)^2 = 1$

$6(x+1)^2 - 6 - 1 = 6(x+1)^2 - 7$

Vertex $= (-1, -7)$

a. Determine, without graphing, whether the function has a minimum value or a maximum value.

b. Find the minimum or maximum value and determine where it occurs.

c. Identify the function's domain and its range.

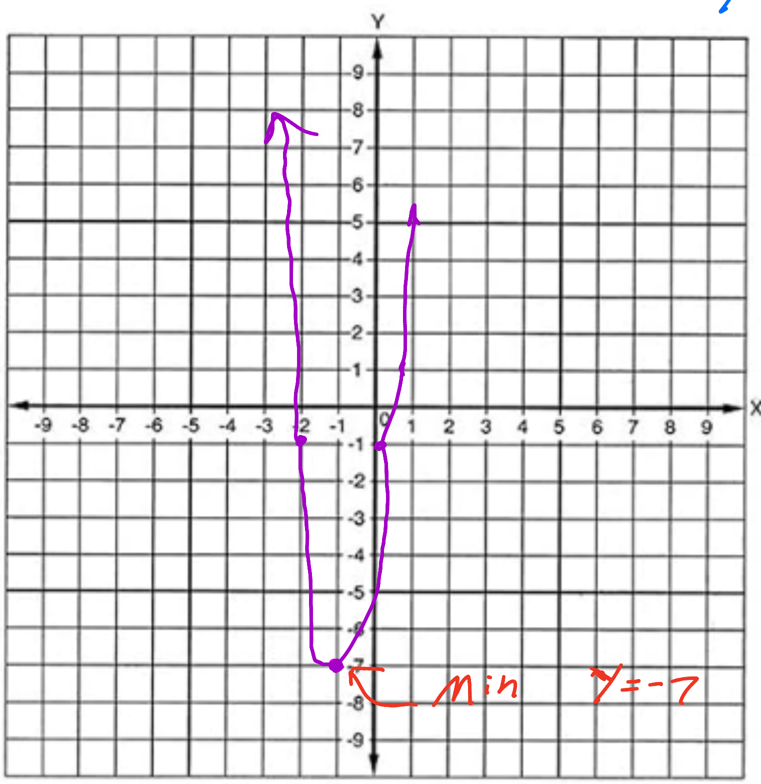
$x = -1$

$y = -7$

Opens up

Scale = 6

$F(x) = 6(x+1)^2 - 7$



x	y
-1	$-7 = 6 \cdot 0^2 - 7$
0	$-1 = 6(1)^2 - 7$
-2	$-1 = 6(-1)^2 - 7$
1	$17 = 6(2)^2 - 7$
-3	$17 = 6(-2)^2 - 7$

Opens up Min

Opens down Max

domain (x-values)

$(-\infty, \infty)$ or $x = \mathbb{R}$

Range $[-7, \infty)$

y-values

Write the equation of the following parabola in vertex form.

The vertex is $(\overset{h}{-2}, \overset{k}{-3})$ and the graph passes through the point $(0, -1)$.

$y = \pm a(x-h)^2 + k$

$y = \pm a(x-(-2))^2 + -3$

$y = \pm a(x+2)^2 - 3$

opens up = +

Opens up

$y = a(x+2)^2 - 3$

Through $(0,-1)$

$y = a(0+2)^2 - 3 = -1$

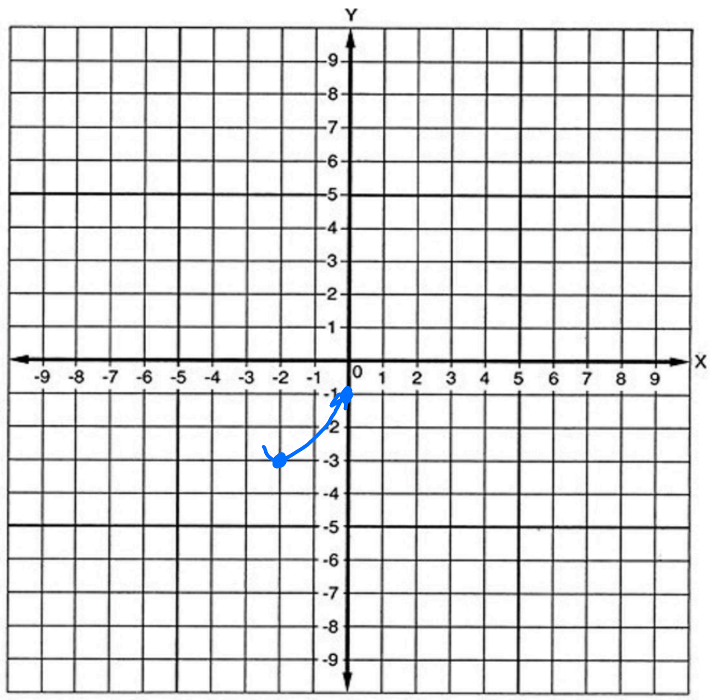
$-1 = 4a - 3$

$+3 \qquad +3$

$2 = 4a$

$\frac{1}{2} = a$

$y = \frac{1}{2}(x+2)^2 - 3$



Evaluate $\dfrac{x^2+17}{6-x}$ for $x = 4i$.

$$\frac{x^2+17}{6-x} = \frac{(4i)^2+17}{6-4i} = \frac{16\cdot i^2+17}{6-4i} = \frac{16(-1)+17}{6-4i} = \frac{-16+17}{6-4i} = \frac{1}{(6-4i)}\frac{(6+4i)}{(6+4i)}$$

$$\frac{6+4i}{36 \cancel{+24i} \cancel{-24i} - 16i^2} = \frac{6+4i}{36-16(-1)} = \frac{6+4i}{36+16} = \frac{6+4i}{52} = \frac{6}{52} + \frac{4i}{52} = \frac{\cancel{2}\cdot 3}{\cancel{2}\cdot 26} + \frac{\cancel{2}\cdot\cancel{2}\, i}{\cancel{2}\cdot\cancel{2}\cdot 13}$$

$$\frac{3}{26} + \frac{i}{13}$$

Perform the indicated operations and write the result in standard form.

$$\frac{3}{(3+i)(6-i)} = \frac{3}{18 \underbrace{-3i+6i}_{+3i} - i^2} = \frac{3}{18+3i-(-1)} = \frac{3}{18+3i+1} = \frac{3}{(19+3i)}\frac{(19-3i)}{(19-3i)}$$

$$\frac{57-9i}{361 \cancel{-57i} \cancel{+57i} - 9i^2} = \frac{57-9i}{361-9(-1)} = \frac{57-9i}{361+9} = \frac{57-9i}{370} = \frac{57-9i}{2\cdot 5\cdot 37} = \frac{57}{370} - \frac{9i}{370}$$

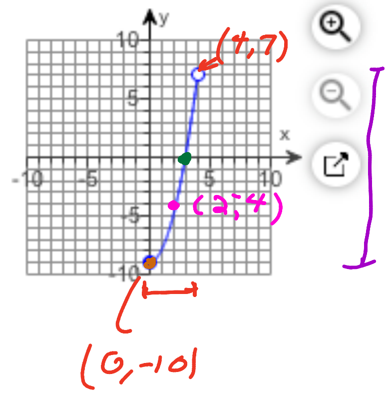
Perform the indicated operations and write the result in standard form.

$$\frac{8}{2+\frac{5}{i}} = \frac{8}{\frac{2i}{1\cdot i} + \frac{5}{i}} = \frac{8}{\frac{2i+5}{i}} = \frac{\frac{8}{1}}{\frac{2i+5}{i}} = \frac{8}{1}\cdot\frac{i}{2i+5} = \frac{8i}{(2i+5)}\frac{(2i-5)}{(2i-5)}$$

$$\frac{16i^2-40i}{4i^2 \cancel{-10i} \cancel{+10i} - 25} = \frac{16(-1)-40i}{4(-1)-25} = \frac{-16-40i}{-4-25} = \frac{(-16-40i)\cdot -1}{-29\cdot -1} = \frac{16}{29} + \frac{40}{29}i$$

Use the graph to determine the following.

a. the function's domain x values ⇒ $0 \le x < 4$
b. the function's range y values ⇒ $-10 \le y < 7$
c. the x-intercepts, if any $(3,0)$
d. the y-intercept, if any $(0,-10)$
e. the function value f(2) $(2,-4)$



Multiply.

$(2+6i)^2 = (2+6i)(2+6i) = 4 + 12i + 12i + 36i^2 = 4 + 24i + 36(-1)$

$4 + 24i - 36$

$-32 + 24i$

Perform the indicated operation and write the result in standard form.

$\sqrt{-4} - \sqrt{-25}$

$2i - 5i = -3i$

Perform the indicated operations and write the result in standard form, a + b*i*.

$6\sqrt{-49} + 3\sqrt{-81}$

$6 \cdot 7i + 3 \cdot 9i = 42i + 27i = 69i$

Evaluate $x^2 - 2x + 8$ for $x = 3 + i$.

$(3+i)^2 = (3+i)(3+i)$

$9 + 3i + 3i + i^2$

$9 + 6i - 1 = 8 + 6i$

$(3+i)^2 - 2(3+i) + 8$

$8 + 6i - 6 - 2i + 8 = 10 + 4i$

Divide and express the result in standard form.

$$\frac{2}{4-i}\cdot\frac{(4+i)}{(4+i)} = \frac{8+2i}{16+\cancel{4i}-\cancel{4i}-i^2} = \frac{8+2i}{16-(-1)} = \frac{8+2i}{16+1} = \frac{8+2i}{17}$$

Perform the indicated operations and write the result in standard form.

$$\left(11\sqrt{-4}\right)\left(-6\sqrt{-5}\right) = (11\cdot 2i)(-6\cdot i\sqrt{5}) = 22i(-6i\sqrt{5}) = -132i\sqrt{5}$$

Perform the indicated operations and write the result in standard form, a + b*i*.

$(3+i)^2 - (7-i)^2$

$9 + 6i + i^2$

$8 + 6i - (49 - 14i - 1)$

$8 + 6i - (48 - 14i)$

$8 + 6i - 48 + 14i$

$-40 + 20i$

$(3+i)^2 = (3+i)(3+i)$

$9 + 3i + 3i + i^2 = 9 + 6i + - 1 = 8 + 6i$

Perform the indicated operations and write the result in standard form.

$$\frac{-8+\sqrt{-20}}{24} = \frac{-8+\sqrt{-1\cdot2\cdot2\cdot5}}{24} = \frac{-8+\sqrt{i\cdot i\cdot2\cdot2\cdot5}}{24}$$

20
5 4
2 2

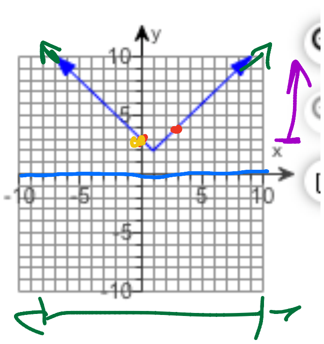
$$\frac{-8+2i\sqrt{5}}{24} = \frac{-8}{24}+\frac{2i\sqrt{5}}{24} = -\frac{1}{3}+\frac{i\sqrt{5}}{12}$$

$$= -\frac{1}{3}+\frac{\sqrt{5}}{12}i$$

Use the graph to determine the following.

a. the function's domain x values $-\infty < x < \infty$ $x = \mathbb{R}$
b. the function's range y-values $2 \le y < \infty$ $[2, \infty)$
c. the x-intercepts, if any Never Hits X-axis
d. the y-intercept, if any (0,3)
e. the function values f(0) and f(3) $F(0)=3$ $F(3)=4$

Assume that the graph of the function continues its trend beyond the displayed coordinate grid.



Perform the indicated operations.

$$(-5+\sqrt{-9})^2 = (-5+3i)^2 = (-5+3i)(-5+3i)$$

$$25-15i-15i+9i^2 = 25-30i+9(-1) = 25-30i-9$$

$$16-30i$$

Use the graph of $y = f(x)$ to graph the function $g(x) = \underbrace{f(x-2)}_{\text{Right 2}} + 1$. ← UP 1